

# Why is order flow so persistent?

Bence Tóth<sup>1a,b</sup>, Imon Palit<sup>2c</sup>, Fabrizio Lillo<sup>3b,d,e</sup>, J. Doyne Farmer<sup>4f,b</sup>

<sup>a</sup>*Capital Fund Management, 23/25, rue de l'Université 75007 Paris, France*

<sup>b</sup>*Santa Fe Institute, 1399 Hyde Park Rd., Santa Fe NM 87501, USA*

<sup>c</sup>*Department of Accounting and Finance, Monash Univeristy, Melbourne, Australia*

<sup>d</sup>*Dipartimento di Fisica, Università di Palermo, Palermo, Italy*

<sup>e</sup>*Scuola Normale Superiore di Pisa, Piazza dei Cavalieri 7, 56126 Pisa, Italy*

<sup>f</sup>*Institute for New Economic Thinking at the Oxford Martin School and Mathematical Institute, 24-29 St. Giles, Oxford OX1 3LB*

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## Abstract

Order flow in equity markets is remarkably persistent in the sense that order signs (to buy or sell) are positively autocorrelated out to time lags of tens of thousands of orders, covering multiple days. Two possible explanations are herding, corresponding to positive correlation in the behavior of different investors, or order splitting, corresponding to positive autocorrelation in the behavior of single investors. We develop a method to distinguish these behaviors, apply it to data from the London Stock Exchange, and show that at the level of single investors, the persistence of order flow is overwhelmingly due to splitting rather than herding.

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**Keywords:** Market microstructure; Order flow; Herding; Order splitting; Price impact; Behavioral finance

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<sup>1</sup>Corresponding author. E-mail: ecneb.htot@gmail.com. Tel.: +33 1 49 49 59 49

<sup>2</sup>E-mail: imon.palit@monash.edu

<sup>3</sup>E-mail: fabrizio.lillo@sns.it

<sup>4</sup>E-mail: doynefarmer@gmail.com

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## I. Introduction

Order flow in equity markets is persistent in the sense that orders to buy tend to be followed by more orders to buy and orders to sell tend to be followed by more orders to sell. Positive serial autocorrelation for the first autocorrelation of signed order flow has been observed in many different markets<sup>5</sup>. In fact, order flow is *remarkably persistent*: As illustrated in Figure 1, all the coefficients of the autocorrelation function of signed order flow are positive out to large lags, corresponding in trade time to tens of thousands of transactions or in real time to many days<sup>6</sup>. This is highly consistent across different markets, stocks, and time periods.

In this paper we perform an empirical study to elucidate the cause of this remarkable persistence. We distinguish two basic mechanisms:

1. *Order splitting* by single investors. This has a natural strategic motivation: To reduce impact investors split their orders into smaller pieces and execute them gradually, as originally shown by Kyle (1985)<sup>7</sup>.
2. *Herding*, i.e. similar behavior of different investors. Herding can arise from either psychological or rational motives<sup>8</sup>, and herds can be directed from outside or emerge from within through the interactions of individual investors. Here we study examples of both types of herding, corresponding to public versus private information:

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<sup>5</sup>Positive autocorrelation for a single lag was observed in the Paris Bourse by Biais, Hillion and Spatt (1995), in foreign exchange markets by Danielsson and Payne (2001), and in the NYSE by Ellul et al. (2007) and Yeo (2008).

<sup>6</sup>The extreme persistence of order flow was independently pointed out by Bouchaud et al. (2004) and Lillo and Farmer (2004). In fact order flow is so persistent that it is a long-memory process, i.e. its autocorrelation function is non-integrable (Beran, 1994). This has been shown for the London and New York stock exchanges by Lillo and Farmer (2004), for the Paris stock exchange by Bouchaud et al. (2004) and for the Spanish stock exchange by Vaglica et al. (2008) and Moro et al. (2009). Note that Axioglou and Skouras (2011) argue that order flow is much more persistent within a given day than across successive days. None of our results here depend on long-memory; we mention this only to emphasize the extreme persistence of order flow.

<sup>7</sup>A model to explain the extreme persistence in equity order flow in terms of order splitting was made by Lillo, Mike, and Farmer (2005); see also Farmer et al. (2011) and Toth et al. (2011).

<sup>8</sup>There is a large literature on herding. See for example (Banerjee, 1992, 1993; Bikhchandani, Hirshleifer, and Welch, 1992; Orlean, 1995; Raafat, Chater, and Frith, 2009; Barber, Odean, and Zhu, 2009). Several previous studies have focused on the effect of communication network structure on price fluctuations; see (Kirman and Teyssiere, 2002; Iori, 2002; Cont and Bouchaud, 2000; Tedeschi, Iori, and Gallegati, 2009). Lakonishok, Shleifer, and Vishny (1992) and Wermers (1999) find only weak evidence for herding. Lebaron and Yamamoto (2007; 2008) constructed an agent-based model of imitation and demonstrated that it could produce long-memory in order flow.

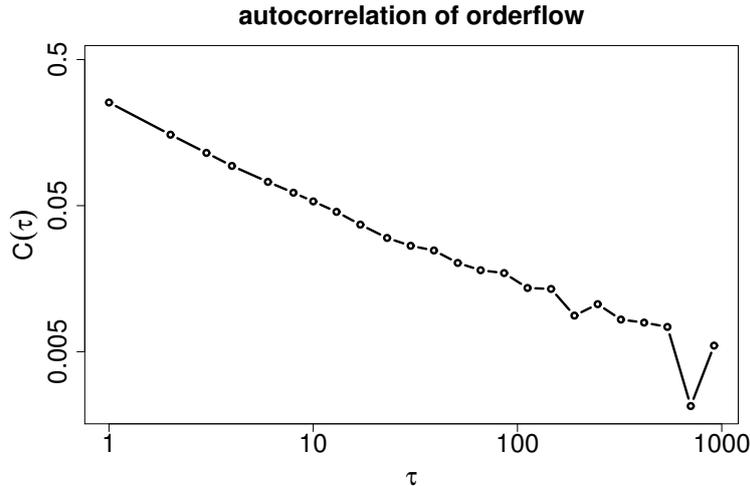


Figure 1: Autocorrelation function of order flow for the stock AZN in the first half of 2009, plotted on double logarithmic scale. The time lag  $\tau$  is measured in terms of number of effective market order placements, where an effective market order is any order that results in an immediate transaction. To reduce estimation error we use order signs rather than order size and individual autocorrelations are binned for large lags. The results are similar if we use order volume or if we include non-crossing limit orders as well as effective market orders.

- (a) *Lagged response to public information.* In this case autocorrelations in order flow come about because different investors respond to public information in a similar way, but with differences in response time. Persistence in order flow can simply reflect persistence in public information arrival or it can be caused by diversity in response times.
- (b) *Imitation triggered by private information.* In this case investors observe each other, directly or indirectly, and react to private information. If one investor buys, other investors are more likely to buy. Even if private information arrival is IID, the time dynamics of the response can make order flow persistent.

Our goal in this paper is study the behavior of *single investors*, by which we mean trading entities with a coherent trading strategy. This could correspond to a specific trading account within an institution such as an investment bank or hedge fund, or a private individual trading for his or her own account. To determine the cause of persistence in order flow we develop a method that uses identity information about orders to decompose the autocorrelation of order flow into herding versus splitting components. If we were able to observe single investors this would allow us to unambiguously distinguish splitting versus herding behavior.

Unfortunately, however, we are not able to observe single investors. What we do have is a unique data set from the London Stock Exchange (LSE) with codes indicating

the exchange member who executed each order. Members of the exchange may trade for their own accounts, but they may also act as brokers for investors who are not members of the exchange<sup>9</sup>. We thus have the problem that while we have potentially useful information about identity, it is not fine-grained enough to allow us to observe single investors, and our results are potentially difficult to interpret.

We cope with this problem by developing several different null models for the behavior of single investors and also developing several different null models for the way in which single investors choose brokers. The null models for single investors are specific instantiations of the mechanisms for splitting and herding discussed above. We study three different models for brokerage. In the first we assume a single investor randomly chooses a broker and always uses that same broker; in the second we assume single investors randomly choose a (possibly new) broker for each trade. The third is a joint null model that assumes that single investors belong to a social network, which similarly influences their trades and their brokerage choice. We then compute what would be observed using brokerage data under a variety of different joint null model for single investor behavior and brokerage choice and compare them to what we actually observe in the data.

This approach might seem unlikely to succeed, for many reasons. A given brokerage may aggregate the trades for thousands of single investors. Furthermore, there is significant heterogeneity across LSE member firms; some act solely as brokers, others as market makers, others as proprietary traders. The nature of the financial intermediation they provide is very different, they use a variety of different trading algorithms, and they likely have very different motives. Thus *a priori* it might seem unlikely that an unambiguous result would emerge.

Surprisingly, we do get unambiguous results. For timescales of a few hours or less, where we have enough data for good statistical significance, we are able to show that at the level of single investors, order splitting completely dominates herding. In contrast the contribution of herding is negligible, except at very short timescales. The reason we are able to get unambiguous results is because null models such as dynamically random brokerage result in behavior that looks like herding, whereas from the empirical analysis at the level of exchange members it is clear that splitting is overwhelmingly favored over herding<sup>10</sup>.

Our results here also have other interesting implications. When we compare the data for different exchange members we find that for the largest members the behavior is surprisingly homogenous: thirteen out of the fifteen largest members all have more or less identical correlations in their order flow. Another interesting conclusion from

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<sup>9</sup>We often use the terms “member” and “broker” interchangeably.

<sup>10</sup>Since obtaining the results presented here one of us (FL) has verified the main conclusions of the paper using a unique dataset for the Russian stock market which contains agent-level identifiers.

this work is that we are able to place bounds on the behavior of investors in choosing brokers. In particular our analysis makes it clear that investors work with only a subset of brokers, and that dynamical variation in the choice of brokers is limited.

Our results here add to earlier evidence that order splitting is an important effect. Early empirical studies by Chan and Lakonishok (1993, 1995) using brokerage data with information about single investors showed that order splitting is widespread. Chordia, Roll, and Subrahmanyam (2002, 2005) found that daily order imbalances are serially autocorrelated and highly persistent<sup>11</sup>. Lee, Liu, Roll, and Subrahmanyam (2004) compared daily autocorrelations in imbalances with and without the consecutive trades by the same agent. They investigated the role played by order splitting and herding-induced imbalance persistence. They found evidence of both herding and splitting phenomena with foreign and small institutions engaging in order splitting, small foreign investors engaging in both herding and order-splitting and domestic institutions engaging in herding behavior. Lillo, Mike and Farmer (2005) introduced a model for order splitting connecting the size of large orders with the autocorrelation of order flow and showed that its predictions gave good agreement with data from the London Stock Exchange. Gerig (2007), demonstrated that for the LSE stock AZN the trades coming from the same brokerage have long-memory, whereas trades from different brokerages do not (see also Bouchaud, Farmer, and Lillo (2009)). Vaglica et al. (2008) reconstructed the size of large orders from brokerage data and found that it is distributed as predicted by Lillo, Mike and Farmer. The work presented here makes the evidence much stronger and more quantitative by developing precise tests for splitting v.s. herding, and systematically analyzing six stocks over a period of ten years, a sample of more than 39 million orders.

The persistence of order flow and its cause have important economic consequences. Under the herding hypothesis the persistence of order flow is due to slow propagation of information, either slow reaction to public information or slow diffusion of private information, so that information is only gradually incorporated into prices. In contrast, order splitting suggests that individuals receive private information (which in order to be new must be uncorrelated with past information arrival) and only trade gradually for strategic reasons. As with herding, information is only slowly incorporated into prices, but for very different reasons. Order splitting in the strong form that we observe here

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<sup>11</sup>Order imbalance for each stock over any time interval is typically calculated using the difference in the number of buy market orders and sell market orders, or the difference in the dollar value from buy market orders and sell market orders. Variations of this metric can use either a scaling factor or calculating a ratio instead of a difference. Brown, Walsh, and Yuen (1997), Chordia, Roll, and Subrahmanyam (2002), Chordia and Subrahmanyam (2004), Chordia, Roll, and Subrahmanyam (2005), Lo and Coggins (2006), Boehmer and Wu (2006) found correlation between order imbalance and returns, however, the evidences are not consistent on the direction of causality. Chordia, Roll, and Subrahmanyam (2008) looking at 5 minute order imbalances have found that return predictability has declined over time.

implies that the market deviates substantially from the equilibrium that would prevail if all participants were forced to reveal their true intentions at the outset of trading.

The persistence of order flow also has important economic consequences because it places strong constraints on the interaction of order flow and prices, which has implications for market impact<sup>12</sup>. While the individual autocorrelation coefficients of order flow at each lag are small, the fact that they are all positive implies substantial predictability over long time horizons<sup>13</sup>. At the same time, in an efficient market the autocorrelation of price returns must be small. Lillo and Farmer (2004) have argued that under the assumption that market impact is permanent this implies that liquidity is asymmetric between buying and selling in a time varying manner; Bouchaud et al. (2004; 2006) et al. have argued that this can also be resolved if impact is completely temporary, albeit decaying extremely slowly<sup>14</sup>.

This affects the functional form of market impact, i.e. the dependence of price changes on order size. Huberman and Stanzl (2004) have shown that if liquidity is constant then permanent impact must be linear to avoid arbitrage; however, when liquidity is time varying, other functional forms become possible, see Gatheral (2010). The results cited above show that liquidity is indeed time varying, and empirical studies suggest that market impact is concave. Under the assumption that trades are bundled by brokers and executed by order splitting algorithms, Farmer et al. (2011) show that the autocorrelation function of order flow determines the functional form of the market impact of large institutional orders, including both its permanent and temporary components. Their derivation does not apply if the persistence of order flow is due to herding. Thus our results here provide important background information relating to the nature of market impact.

The rest of this paper is organized as follows: Section II describes the data. In Section III we derive a decomposition of the autocorrelation function into splitting and herding components, and present a single empirical result to illustrate how strongly splitting dominates over herding. In Section IV we investigate the question of what results based on brokerage codes imply about single investors. We develop null models of herding and splitting by single investors and null models for brokerage, and show that despite the fact that brokerage distorts the results, it is still possible to infer that splitting at the level of single investors is the overwhelming cause of order flow persistence. In Section V we study the dynamical behavior of splitting versus herding, and also study the

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<sup>12</sup>For a comprehensive summary see Bouchaud, Farmer, and Lillo (2009).

<sup>13</sup>For an ARMA process predictability decays exponentially in time; for a long-memory process predictability decays as a power law, which is asymptotically much slower.

<sup>14</sup>See also Farmer et al. (2006), Gerig (2007) and Bouchaud, Farmer, and Lillo (2009) for a detailed discussion of the sense in which these are equivalent. More recent work by Eisler, Bouchaud, and Kockelkoren (2011a) shows that when limit orders and cancellations are taken into account these models are not equivalent.

heterogeneity of individual exchange members and show that most of them are extremely persistent in terms of both trade direction and the clustering in time of their trades. In Section VI we investigate the negative contribution of herding to the autocorrelation function more carefully, and show that it is driven by heterogeneity in the response of investors to market orders that change the price, versus those that do not. In the conclusions we summarize and reflect on the economic implications of our results.

## II. Data

This study is performed using data from the London Stock Exchange (LSE). There are two parallel markets, the on-book market (SETS) and the off-book market (SEAQ). In the on-book market trades take place via a fully automatic electronic order matching system, while in the off-book market trades are arranged bilaterally via phone calls. We restrict our study to the on-book market. Note that there are no official market makers, though it is possible for any member firm to act as a market maker by posting bids and offers simultaneously.

We study six stocks in the period from June 2000 to June 2009, with the exception of a six month period from January to May in 2003. We divide the data for each stock into 17 subperiods of six months each, for a total of  $17 \times 6 = 102$  samples. The six stocks we study are AstraZeneca (AZN), BHP Billiton (BLT), British Sky Broadcasting Group (BSY), Lloyds Banking Group (LLOY), Prudential (PRU), Vodafone Group (VOD)<sup>15</sup>. In cases where we present data for only one period we will use AZN in the first half of 2009. In this period the typical number of market orders per day is between 4,000 and 5,000. The number of market orders in each sample ranges from 91,710 to 1,416,000, with an average of 382,500.

Our data set contains all orders placed in the on-book market. The aspect of this data that makes it unique is that each order is characterized by a code identifying the member who placed the order<sup>16</sup>. The number of members varies throughout the sample, but there are typically about 100 members. To avoid data with poor statistics, in each six month period we remove members who make less than 100 transactions across the full period, which typically leaves about 80 active members. The activity level is very heterogeneous. For example, the 5 most active market members are responsible for 40-50% of transactions and the 15 most active ones are responsible for 80-90% of transactions. The value of the Gini coefficient of member activity averaged across the

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<sup>15</sup>We have also investigated other stocks and find similar results.

<sup>16</sup>We do not have the actual names of the members, but rather anonymized codes uniquely identifying each member. For the period 2000-2002 we are only able to track members for one month, because in this part of the data codes are reshuffled each month. Because we are operating on timescales of at most a few hours this does not affect the results.

102 samples is 0.87. This is quite consistent across stocks and time periods, with a standard deviation of only 0.02. Thus the trading activity is strongly concentrated in a relatively small number of member firms.

We have performed the analysis given here in three different ways: (1) market order signs  $\epsilon_t$ , where a market order is defined as any event that results in an immediate transaction; (2) signs of all orders, including both market and limit orders; (3) signed volume  $v_t$  of transactions. The results are similar in all three cases, except that for signed volume they are somewhat noisier. We present only the results for case (1), market order signs. The number of market orders is used to measure time, i.e.  $t' = t + 1$  whenever there is a market order placement.

### III. Decomposing order flow

In this section we present a simple decomposition of the autocorrelation of order flow and present an empirical result illustrating the strong dominance of splitting over herding.

#### 1. Decomposition

Consider a time series of orders of sign  $\epsilon_t$ , where  $\epsilon_t = +1$  for buy orders and  $\epsilon_t = -1$  for sell orders. For convenience we measure time in terms of order arrivals, so that time advances by one unit every time a new order arrives. We define the signed order flow  $\epsilon_t^i$  to be zero if the order at time  $t$  was not placed by investor  $i$  and to be the order sign otherwise, i.e.

$$\begin{aligned}\epsilon_t^i &= 1 \Rightarrow \text{buy order placed by investor } i \\ \epsilon_t^i &= 0 \Rightarrow \text{order placed by another investor} \\ \epsilon_t^i &= -1 \Rightarrow \text{sell order placed by investor } i.\end{aligned}$$

As already mentioned the analysis can be performed using market orders (as we have done here), all orders, or with signed order volumes, with similar results.

Let  $N$  be the length of the time series,  $N^i$  be the number of orders due to investor  $i$  and  $N^{ij}(\tau)$  the number of times that an order from investor  $i$  at time  $t$  is followed by an order from investor  $j$  at time  $t + \tau$ . Similarly, let  $P^i = N^i/N$  be the fraction of orders placed by investor  $i$ , and  $P^{ij}(\tau) = N^{ij}(\tau)/N$  be the fraction of times that an order from investor  $i$  at time  $t$  is followed by an order from investor  $j$  at time  $t + \tau$ . The weighting function  $P^{ij}(\tau)$  captures the extent to which investors  $i$  and  $j$  tend to be active with a given time lag  $\tau$ , regardless of the sign of their orders. If investors act independently then in the large  $N$  limit:

$$P^{ij}(\tau) = P^i P^j \quad \forall i, j,$$

and  $P^{ij}(\tau)$  is independent of  $\tau$ . Deviations from independence can be characterized by

$$\tilde{P}^{ij}(\tau) \equiv P^{ij}(\tau) - P^i P^j. \quad (1)$$

Finally, let the sample mean of the order sign for investor  $i$  be

$$\mu^i \equiv \frac{1}{N^i} \sum_t \epsilon_t^i.$$

The sample autocorrelation in the order signs of agents  $i$  and  $j$  can then be written

$$C^{ij}(\tau) \equiv \frac{1}{N_{ij}(\tau)} \sum_t \epsilon_t^i \epsilon_{t+\tau}^j - \mu^i \mu^j.$$

As shown in the Appendix, the autocorrelation function of order flow can be written

$$C(\tau) = \sum_{i,j} P^{ij}(\tau) C^{ij}(\tau) + \sum_{i,j} \tilde{P}^{ij}(\tau) \mu^i \mu^j. \quad (2)$$

This can trivially be written in the form

$$C(\tau) = C_{split}(\tau) + C_{herd}(\tau), \quad (3)$$

where

$$\begin{aligned} C_{split}(\tau) &\equiv \sum_i P^{ii}(\tau) C^{ii}(\tau) + \sum_i \tilde{P}^{ii}(\tau) (\mu^i)^2, \\ C_{herd}(\tau) &\equiv \sum_{i \neq j} P^{ij}(\tau) C^{ij}(\tau) + \sum_{i \neq j} \tilde{P}^{ij}(\tau) \mu^i \mu^j. \end{aligned} \quad (4)$$

$C_{split}(\tau)$  is the autocorrelation of orders coming from the same investor, and  $C_{herd}(\tau)$  is the autocorrelation of orders coming from different investors.

It is useful to compare the first and second terms of the decomposition in Eq. (2). The first term weights the correlation of order signs by the activity  $P^{ij}(\tau)$ , and the second term weights deviations in activity by the mean order signs  $\mu^i$  and  $\mu^j$ . Since buying and selling roughly balance, for large samples  $\mu^i$  is small and the second term is negligible<sup>17</sup>. Typical values for our dataset are  $|\mu^i| = 0.03$  and  $\sum_{i,j} \tilde{P}^{ij}(1) \mu^i \mu^j = 10^{-4}$ .

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<sup>17</sup>It is not surprising that this term is small – in the long-run one expects  $\mu^i$  to be close to zero. If  $\epsilon_t^i$  represents signed trading volume, for example, then  $\sum_t \epsilon_t^i$  is the total inventory change during the period of the study. If the inventory remains bounded, then  $\mu^i \rightarrow 0$  in the limit  $N \rightarrow \infty$ . Even if  $\epsilon_t^i$  represents order signs, then if investors buy as often as they sell,  $\mu^i$  is small.

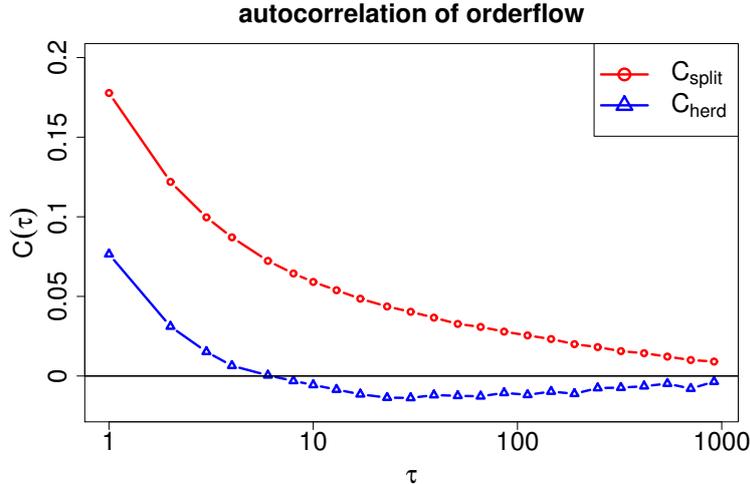


Figure 2: Decomposition of the autocorrelation – A decomposition of the autocorrelation into  $C(\tau) = C_{split}(\tau) + C_{herd}(\tau)$  according to Eq. (3); the horizontal axis is plotted on logarithmic scale and the vertical axis on linear scale. Splitting dominates at all lags. Note the negativity of the herding component for  $\tau > 10$ . Here we show AZN in the first half of 2009, but all of the 102 samples look similar.

Thus the first term of Eq. (2) is three orders of magnitude larger than the second one, and it is a very good approximation to take

$$C(\tau) \simeq \sum_{i,j} P^{ij}(\tau) C^{ij}(\tau). \quad (5)$$

Similar approximations can be used for  $C_{split}(\tau)$  and  $C_{herd}(\tau)$ . One can think of the autocorrelation as a product of two terms, one that depends on activity, as reflected by  $P^{ij}(\tau)$ , and the other that depends on trading direction, as reflected by  $C^{ij}(\tau)$ .

We have thus decomposed order flow in two different ways: (1) Contributions due to trading direction (buying versus selling) versus activity clustering (when investors make their trades), and (2) contributions due to herding versus order splitting, i.e. correlations of different investors versus autocorrelation of the same investor.

## 2. Empirical decomposition based on exchange member data

To illustrate the application of this decomposition we present an empirical result based on order flow data with exchange membership identifiers. Figure 2 shows an example of the decomposition of the autocorrelation function into  $C(\tau) = C_{split}(\tau) + C_{herd}(\tau)$  according to Eq. 3. The splitting term is always positive and is substantially larger than the herding term at all lags. This illustrates our central result, which is that splitting strongly dominates herding.

In Section V we return to investigate the question of herding vs. splitting using exchange member data in much greater detail. Before we do that, however, we introduce the null models we will use to argue that our results also apply at the level of single investors.

#### IV. How can we infer results about single investors from brokerage data?

As described in the introduction, most of the member firms in the LSE are at least partially acting as brokers, lumping together orders from many single investors. Furthermore, single investors may be using multiple brokers, or varying their choice of broker. Given this, how can we ever hope to make inferences about single investors based on data that only identifies exchange members?

We confront this problem by developing a set of null models and investigating how the results are altered when one goes from an analysis when the identity of single investors is known to one in which only the brokers are known. This is done by making explicit models of single investor behavior and explicitly modeling brokerage. We formalize the notion of a brokerage map assigning investors to brokers, and study how brokerage distorts the decomposition of order flow. It is then possible to compare the behavior observed in the data to a set of joint null models combining different scenarios for single investor behavior and brokerage. This provides a reference point for our empirical results. The conclusion is that brokerage can introduce substantial distortions, but they are asymmetric in the sense that splitting is distorted differently than herding. If herding dominated in the real data, we would not be able to make firm conclusions, but since splitting dominates, our results are in the end unambiguous. Even in the hypothetical situation in which the correlations between single investor trading and choice of brokerage are extremely strong it is still not possible to explain the persistence of the autocorrelation of order flow based on herding by single investors.

We first formulate three models for single investor trading behavior, then we formulate two models for brokerage, and then finally a joint model in which the social network that determines trading is correlated with the choice of brokerage. In each case we derive what happens to the decomposition of the autocorrelation of order flow under each possible combination of investor behavior and brokerage.

##### *1. Three models for persistence in order flow*

We now formulate explicit models of splitting and herding. The model of splitting is completely straightforward, but there are many ways to model herding. We have defined “herding” as any form of synchronized trading between different individuals that causes positive correlations in order flow. As discussed in the introduction, we distinguish herding based on a common response to public information from herding

based on the endogenous dynamics of investors who are responding to each others' private information; we provide an example of each. There are of course many variations in how one can define such models; we make some arbitrary choices, aiming for simplicity.

### 1.1. Order splitting

Under order splitting the autocorrelation of order flow is generated entirely by investors who split large trades into small pieces and execute them incrementally. Under the assumption that all persistence is due to order splitting, we assume that information is private and IID. Since by assumption the autocorrelation between different investors is zero, the persistence is entirely due to splitting, and

$$\begin{aligned} C_{split}(\tau) &= C(\tau), \\ C_{herd}(\tau) &= 0. \end{aligned} \tag{6}$$

### 1.2. Herding model I: Public information

Our first herding model produces herding behavior via a similar response to a common external cause – the herd is directed from outside. Assume a public information signal  $I_t$ , which is a random real-valued variable with positive and negative values equally likely. Measure the time  $t$  in terms of order arrival, so that every time an order arrives,  $t \rightarrow t + 1$ . Let there be  $M$  investors, where the trading frequency of investor  $i$  is  $P^i > 0$ , normalized so that  $\sum_{i=1}^M P^i = 1$ . An information signal  $I_t$  arrives at time  $t$  and remains fixed until time  $t + n_t$ , where  $n_t = f(|I_t|)$  and  $f$  is a non-decreasing positive function that takes on only integer values (more information implies more order arrival); the  $n_t$  orders all have the sign of  $I_t$ . The investor placing each order is drawn according to that investor's trading frequency  $P^i$ . We assume sampling with replacement, so that a given investor may be drawn more than once. At time  $t + n_t$  a new information signal  $I_{t+n_t}$  arrives and the process is repeated<sup>18</sup>.

Persistence in order flow can result from several different effects: (1) autocorrelations in  $I_t$ , (2) heavy tails in the distribution  $P(I_t)$ , (3) generalizing the model so that  $n_t = f + \xi_t$ , where  $\xi_t$  is random, or (4) any combination of these three effects. For definiteness we construct the model so that the persistence is entirely driven by effect (1), but we have investigated all of the possibilities above and find that providing we hold the autocorrelation of order flow fixed the choice makes no difference in its decomposition.

The decomposition of the autocorrelation for this model can be computed in closed form. While one might naively assume that a herding model will produce a pure de-

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<sup>18</sup>As stated this model generates  $n_t$  strings of sequential orders all of the same sign. It is possible to make the order flow look more realistic by injecting orders with a random sign, but the only effect is to decrease the prefactor of the autocorrelation without otherwise affecting its time dependence.

composition with all its weight on the herding term, this is not the case. Instead, the relative size of  $C_{split}$  and  $C_{herd}$  depends on the number of investors  $M$  and the variance of their trading frequencies  $P^i$ . As shown in the Appendix,  $C_{split}$  and  $C_{herd}$  are

$$\begin{aligned} C_{split}(\tau) &= C(\tau) \left( \frac{1}{M} + M \text{Var}[P] \right) \\ C_{herd}(\tau) &= C(\tau) \left( \frac{M-1}{M} - M \text{Var}[P] \right). \end{aligned} \quad (7)$$

where  $\text{Var}[P]$  is the cross-sectional variance of investor trading volumes, i.e.

$$\text{Var}[P] = \frac{1}{M} \sum_i (P^i)^2 - \frac{1}{M^2}. \quad (8)$$

The variance lies in the range  $0 \leq \text{Var}[P] \leq \frac{1}{M} \left(1 - \frac{1}{M}\right)$ . The bounds occur when:

- *All investors are equally active,  $\text{Var}[P] = 0$ .*

$$\begin{aligned} C_{split} &= \frac{C(\tau)}{M} \\ C_{herd} &= \frac{M-1}{M} C(\tau). \end{aligned}$$

- *All trading is concentrated in one investor,  $\text{Var}[P] = \frac{1}{M} \left(1 - \frac{1}{M}\right)$ .*

$$\begin{aligned} C_{split} &= C(\tau) \\ C_{herd} &= 0. \end{aligned}$$

If  $C(\tau) > 0$  then  $C_{split}$  is always positive and  $C_{herd}$  is always non-negative. As the trading goes from uniformly distributed to concentrated in a single member,  $C_{split}/C$  grows from 0 to 1 while  $C_{herd}/C$  decreases from 1 to 0. Thus the heterogeneity in investor's trading frequencies controls the relative importance of the herding versus splitting components of the autocorrelation function. This behaves in the way that one would expect: When all investors are equally active, for  $M$  large we see a strong herding contribution and almost no splitting contribution, whereas when there is only one investor active the order flow is indistinguishable from splitting.

### 1.3. Herding model II: Private information with imitation

Our second herding model produces herding endogenously without need for an external coordination mechanism. In this model the investors herd by imitating each other. We assume that investors exist within a social network where information is transmitted

between neighbors. The transmission of information depends on the topology of the network, which also determines the autocorrelation of order flow.

The nodes of the social network correspond to investors and links correspond to influence. The network is connected, so that all nodes are joined to the same graph by at least one link. Investor  $i$  has a binary state  $s_t^i = \pm 1$  that indicates whether, all else equal, this investor prefers to buy or sell. Whether or not the investor actually buys or sells also depends on what her neighbors do. The dynamics are characterized by a parameter  $p \in [0, 1]$  that describes the degree of imitation.

The order flow is generated using the following algorithm: At time  $t$  an investor  $i$  is chosen randomly and an order with sign  $s_t^i$  is submitted. Then  $t \rightarrow t + 1$  and each neighbor  $j$  of investor  $i$  is considered. With probability  $p$  the neighbor takes on the sign of her neighbor, and with probability  $1 - p$  she keeps her sign, i.e. with probability  $p$  investor  $j$  submits an order  $\epsilon_t^j = s_t^i$  and changes state to  $s_{t+1}^j = s_t^i$ , and with probability  $1 - p$  investor  $j$  submits an order  $s_t^j$  and  $s_{t+1}^j = s_t^j$ . The time is once again incremented and the next neighbor, if any, is considered. Once all the neighbors of investor  $i$  have been considered a new node  $i'$  is chosen at random and the process continues.

There is no need for exogenous information in this model. Providing the system is initially placed in a sufficient diverse initial configuration, it will generate time correlated but otherwise random order flow. In the noiseless version this model has two fixed points where all the agents are found in the same state (buy or sell). If the network is large enough these fixed points cannot be reached in a reasonable time. It is also possible to generalize the model to inject external information by occasionally randomly altering the sign of the states of one of the nodes. Providing this is not done too often, however, it does not substantially alter the characteristics of the order flow. The degree distribution  $\mathcal{P}(\ell)$  and the parameter  $p$  determine the autocorrelation of the order flow<sup>19</sup>.

For this model are not able to compute the distortion of the decomposition in closed form. Instead we simulate it and compare it to the public information herding model, holding the autocorrelation function  $C(\tau)$  constant. We construct a social network using preferential attachment (Simon, 1955; Barabasi, Albert, and Jeong, 1999). An initial node is created, and then new nodes are incrementally generated and connected to a randomly chosen pre-existing node with a probability of attachment proportional to its degree. Because we connect to only one pre-existing node at a time the resulting graph is a tree<sup>20</sup>. Simulations presented in the next section show that. Eq. (7) is a good approximation for the order flow decomposition.

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<sup>19</sup>The degree of a node of a graph is the number of links connected to that node

<sup>20</sup>We could more realistically attach to multiple pre-existing nodes, but this does not affect the results.

## 2. Models of brokerage

In this section we address the distortion caused by observing brokerage codes rather than single investors. We introduce the concept of a brokerage map, present examples of two possible brokerage maps, and study how the order flow decomposition of the single investor models introduced in the previous section are altered by each of these two brokerage maps. The two types of brokerage maps we consider are fixed random brokerage and dynamically random brokerage. In the next section we generalize the private information imitation model to allow the social network to determine both order submission and choice of broker.

### 2.1. Fixed random brokerage

This model assumes that each investor randomly chooses a single broker and thereafter always executes through that broker only. We constrain the random choices to match the empirically observed brokerage trading frequency  $P'^i$ . Mirroring the same notation that we used for investors, assume there are  $M'$  brokers labeled by an index  $i$  and that  $P'^i$  is the trading frequency of broker  $i$ .

Under the fixed random brokerage map the order flow decomposition of the three investor models presented in the previous section is distorted as follows:

- *Order splitting.* The decomposition is unchanged, i.e.  $C_{split}(\tau) = C(\tau)$  and  $C_{herd} = 0$ . This is true regardless of  $P'^i$ .
- *Public information herding model.* The decomposition is given by Eq. (7), except that  $M = M'$  and  $\text{Var}[P] = \text{Var}[P']$ . Thus, when order flow is spread across many brokerages with similar trading volume the result is dominated by herding, but if it is concentrated into a small number of brokerages the splitting component increases<sup>21</sup>.
- *Private information herding model.* We build an investor social network as described in the previous section, and then construct a fixed random brokerage map. We assume  $M' = 50$  brokers and match their trading frequency to the 50 largest exchange members for *AZN* during the first half of 2009. The imitation frequency is chosen to be  $p = 0.9$  with  $M = 10,000$  investors. Simulations are run for  $10^6$  time steps.

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<sup>21</sup>While it might seem surprising that this no longer depends on  $M$  or  $\text{Var}[P]$ , one should bear in mind that under fixed random brokerage we are assuming the correlation between single investors is independent of their broker. Furthermore, the number of investors  $M$  and the variance of their trading frequency  $\text{Var}[P]$  have to be consistent with  $P'^i$ . Since behavior below the level of the brokerage is not observable, the trading of any investor within a given brokerage has the same effect, and the results are as if there were  $M'$  rather than  $M$  investors with variance  $\text{Var}[P']$

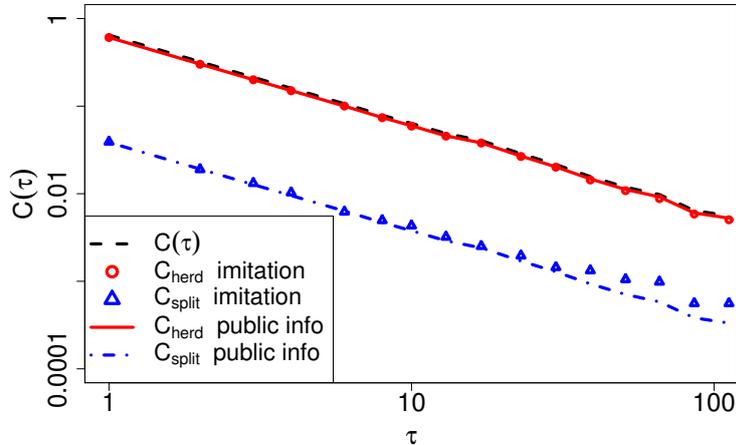


Figure 3: Decomposition of the autocorrelation of order flow for the two herding models using a fixed random brokerage map. For the private information (imitation) model, order flow is generated as described in the text, and the splitting (blue triangles) and herding (red circles) contributions are computed numerically using Eq. (4). For the public information herding model we assume the same total autocorrelation  $C(\tau)$  (black dashed line) as the imitation model, and compute the splitting (blue dots) and herding (red line) contributions using Eq. (7). Broker trading frequencies  $P^i$  are chosen to match AZN in the first half of 2009. Herding strongly dominates splitting and the decomposition for the two models is nearly identical.

The autocorrelation and its decomposition for the two herding models are shown in Figure 3. We simulate the private information model with the parameters above and choose the autocorrelation function  $C(\tau)$  for the public information model to be the same<sup>22</sup>. The resulting decomposition for the two herding models is nearly identical. In both cases the resulting decomposition is completely dominated by the herding component. While there is a non-zero splitting component, it is more than an order of magnitude smaller.

Thus for order splitting the fixed random brokerage map introduces no distortion, whereas for herding models the distortion is strongly dependent on the brokerage trading frequencies. When the trading frequencies are uniform these models strongly favor herding, but as they become concentrated on a few brokerages the splitting component increases.

<sup>22</sup>For the public information herding model the persistence of order flow derives from the heavy tailed degree distribution of the social network. The process of imitation converts the scale free power law degree distribution of the social network, which by construction is of the form  $\mathcal{P}(\ell) \sim \ell^{-\eta+1}$ , into order flow with power law decay of autocorrelations. In fact, in the limit  $p \rightarrow 1$  it can be shown that the autocorrelation function decays asymptotically as  $C(\tau) \sim \tau^{-(\eta-1)}$ .

## 2.2. Dynamically random brokerage

To conceal order flow some investors may vary their choice of brokers. Dynamically random brokerage captures this strategy in its extreme form: On every trade each investor randomly chooses a broker according to the broker trading frequencies  $P'^i$ . There is thus no memory of the past, and no allegiance of any investor to any broker. Random choice implies that  $P'^{ij} = P'^i P'^j$ , and therefore the decomposition is given by Eq. (7). The resulting decomposition is thus identical to that of the public information herding model under a fixed random broker map. This is true regardless of the investor model – the behavior of individual investors makes no difference.

As we saw in the previous section, when we use realistic broker trading frequencies the herding component is much larger than the splitting component. Thus, we have the interesting result that the distortion caused by dynamically random brokerage creates the appearance of herding, regardless of what the investors are actually doing. Given that our empirical results show a strong dominance of splitting over herding, this is serendipitous, as it means that we can state with confidence that investors are not using dynamically random brokerage, or anything close to it.

## 3. Joint model where social network correlates trading and brokerage

The previous models treat trading and brokerage as independent. What if the trading behavior of single investors is correlated with their choice of brokers? To study this question we have generalized the public information herding model to allow for the possibility that neighbors in the influence network also tend to use the same broker.

In the generalized private information model the social network and the brokerage assignments are made in tandem. As we construct the social network, when we attach each new investor to a previous investor, we assign the same broker with probability  $\Phi$  or a random broker with probability  $1 - \Phi$ . As a result there is a correlation between neighborhood relationships in the trading graph and brokerage assignment<sup>23</sup>.

## 4. Comparison of data to investor and brokerage null models

### 4.1. Summary of decomposition implied by joint null models

Figure 4 compares the possible combinations of null models for investor behavior and brokerage maps to the real data. The key parameter that determines our ability to extract information about individual investors from brokerage data is the variance

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<sup>23</sup>An inconvenient feature of this model is that  $\Phi$  and  $P'^i$  are not independent, so it is harder to construct a network with given values of  $P'^i$ . As  $\Phi$  varies from zero to one the trading frequencies of the brokers necessarily become more concentrated.

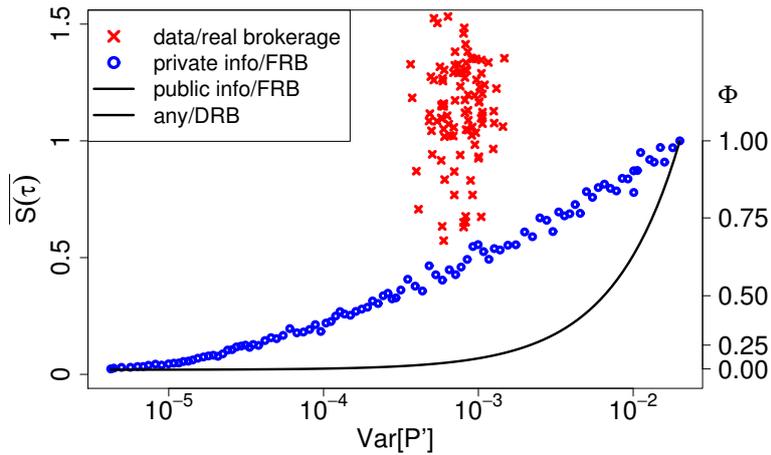


Figure 4: Comparison of the null models for investor behavior and brokerage maps to real data. The splitting decomposition for the 102 real data samples (indicated by red x's) is compared to the predictions of three different null hypotheses. The variance of the brokerage trading frequencies varies along the horizontal axis. On the left the trading frequencies are equal and thus the variance is small, and on the right trading is concentrated in a few brokerages and the variance is large. The left vertical axis indicates the splitting ratio  $S(\tau) = C_{split}(\tau)/C(\tau)$  averaged over  $1 \leq \tau \leq 50$  and the right vertical axis indicates the broker-neighbor correlation  $\Phi$  (for the generalized imitation model only). The null hypotheses are generated by combining a model for investor herding with a model of brokerage. The blue circles represent simulations of the private information herding model (labeled “imitation”) with Fixed Random Brokerage (FRB). The black curve is the public information model (with  $\Phi = 0$ ) and Fixed Random Brokerage; alternatively it represents the prediction of the Dynamically Random Brokerage (DRB) model under any investor model. Under any brokerage model all 102 samples are outside the range predicted by any of the null models that involve herding.

of the trading volume of brokerages,  $\text{Var}[P']$ , which is shown on the horizontal axis of the figure, and is plotted on logarithmic scale. The minimum value  $\text{Var}[P'] = 0$  implies that all the brokerages have the same volume, and the maximum value  $\text{Var}[P'] \approx 1/M$  implies that all trading is concentrated in a single brokerage. The vertical axis on the left plots the fraction of the autocorrelation due to splitting, i.e.

$$S(\tau) = \frac{C_{split}(\tau)}{C(\tau)}. \quad (9)$$

When the splitting ratio is zero herding dominates and when it is one, splitting dominates. For the scenarios constructed here  $S(\tau) = S$  is independent of  $\tau$ .

The behavior of the scenarios developed in this section can be summarized as follows (please refer to Figure 4):

- *Public information herding model/fixed random brokerage* is given by Eq. (7) with  $M = M'$  and  $\text{Var}[P] = \text{Var}[P']$ , and is represented by a solid black line. For small values of  $\text{Var}[P']$ , where the volumes of the brokerages are fairly uniform, the spitting ratio is near zero, i.e. herding dominates. However at larger values of  $\text{Var}[P']$ , where the volumes of the brokerages are very different, the spitting ratio is near one, i.e. splitting dominates. The transition begins at roughly  $\text{Var}[P'] \approx 10^{-3}$ .
- *Any investor model/dynamic random brokerage*. This is also represented by the solid black line.
- *Generalized private information herding model/fixed random brokerage*. This is represented by the blue circles. In this case we also plot the correlation parameter  $\Phi$  on the vertical axis on the right. Since the relation between  $\Phi$  and  $S$  is nonlinear, this has a nonlinear scale.  $S$  increases nearly linearly as a function of  $\text{Var}[P']$ .

In summary, the joint null hypotheses we have developed indicate when splitting and herding can be distinguished at the level of single investors. If  $\text{Var}[P']$  is large it is impossible to infer anything about single investor behavior. If  $\text{Var}[P']$  is sufficiently small, however, then our ability for inference depends on the results. If herding is dominant, then our inferences are ambiguous due to the fact that herding under the scenario where single investors consistently choose a single broker is indistinguishable from the scenario where they choose brokers in a dynamically random fashion (in which case the behavior of single investors is irrelevant to what is observed at the level of brokerages). In contrast, if splitting is dominant, we can rule out the possibility that investors use many different brokers, and we have a clear inference, providing the effect is sufficiently strong.

## 4.2. Comparison to data

To compare to the real data we compute the decomposition of the autocorrelation defined by Equation (4) for each of the 102 samples and compute the splitting ratio  $S(\tau) = C_{split}(\tau)/C(\tau)$  and average it for lags  $1 \leq \tau \leq 50$ . The value for each sample is represented by a red “x” in Figure 4. The real data is well-separated from all of the null models in all 102 cases, indicating that none of them can explain the data, and that splitting is the only possible explanation. The only null model that comes close is the joint null model that assumes that there is a social network causes herding, correlating order placement under private information with broker choice. A few of the samples are close to this null model, but most of them have values of  $S$  that are a factor of two or three greater. Note also that in order to even come close, for these samples it is necessary to assume that the correlation between trading and broker choice is the order of 60%, which seems high.

The brokerage maps we have chosen are extreme and bracket the space of possibilities. We believe we can thus be confident that there are no reasonable circumstances in which the distortion introduced by the use of brokerage data could plausibly be large enough to refute our conclusion that splitting dominates over herding on short time scales.

An interesting side-effect of this work is that the fact that the real data is so far from the dynamically random brokerage null model demonstrates that there must be a substantial degree of consistency in the choice of brokers.

## V. Time dependence and heterogeneity

Now that we have established that brokerage data can be used to make inferences about single investors, we now study the time dependence of the results. We study both the dependence of  $C_{split}(\tau)$  on  $\tau$ , and we also study how this decomposition has evolved during the course of the ten years of data in our data sets. Then we investigate the heterogeneity of the brokers in more detail.

### 1. Splitting and herding as a function of time

The behavior shown in Figure 2 is remarkably consistent across all 102 samples. To illustrate this, in the left panel of Figure 5 we plot the decomposition  $C(\tau) = C_{split}(\tau) + C_{herd}(\tau)$  averaged across all of the 102 samples, and also plot the standard deviation across the samples for each time lag. There is remarkably little variation across the samples. The standard deviations are small compared to the difference between  $C_{split}$  and  $C_{herd}$ . For  $\tau \leq 100$ , where we have the best statistical reliability, there is not a single case in which the herding term is larger than the splitting term.

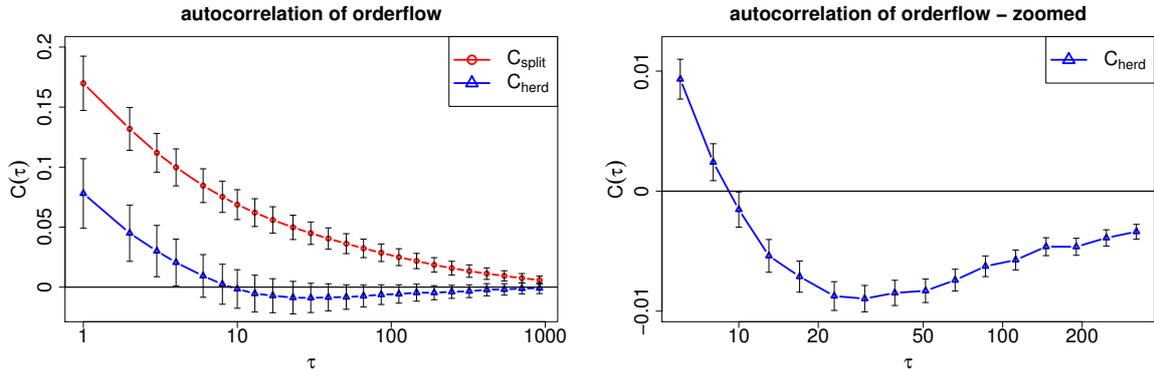


Figure 5: The decomposition of the autocorrelation for the ensemble of stocks illustrates the consistency with which splitting dominates herding. *Left panel:* Autocorrelations of market order signs averaged across all 102 samples (spanning nine years and six stocks). The bars are standard deviations. *Right panel:* The herding component in the left panel is magnified to better observe the negative contribution of the herding component. The bars in this panel are standard errors. For both plots we use logarithmic scale on the horizontal axis and linear scale on the vertical axis.

The negative contribution by the herding component observed for AZN is not special to this stock or this time period: Almost all stocks show similar behavior. To examine this in more detail, in the right panel of Figure 5 we enlarge the scale and plot only  $C_{herd}$ . Instead of showing the standard deviation across the samples, we show the standard error. The fact that the average value of  $C_{herd}$  is consistently negative for  $10 \leq \tau \leq 250$ , at many lags by more than three times the standard error, suggests that this effect is real. We will return in Section VI to perform better statistical tests and shed some light on the cause of this phenomenon.

To have a quantitative comparison of the size of the two terms and to see how this changes with  $\tau$ , in the left panel of Figure 6 we plot the splitting ratio  $S(\tau)$  as a function of  $\tau$ . Note that  $S(\tau)$  is larger than one if  $C_{herd}$  is negative. In the left panel we show results for AZN for the first half of 2009, and in the right panel we show the average result for all 102 samples. For  $\tau = 1$  the splitting accounts for about 75% of the autocorrelation and herding explains about 25%. For larger lags splitting becomes relatively even more important. For lags larger than roughly 10,  $S(\tau)$  becomes larger than one because the herding part becomes negative, and  $S(\tau)$  rises to about 1.5. This means that the splitting term is about 2.5 times the absolute value of the (anti)herding term. For large  $\tau$  the standard deviation across the samples becomes large because  $C_{split}$  and particularly  $C_{herd}$  are small, and we are taking ratios of small numbers.

As a point of comparison in Figure 6 we compare the real data to the predictions of the herding null model given in Eq. (7) with parameters matched to the data (or equivalently to dynamically random brokerage with any investor model). For the data  $S(\tau) = C_{split}(\tau)/C(\tau)$  increases from about 0.7 to 2 with increasing  $\tau$ , in contrast to

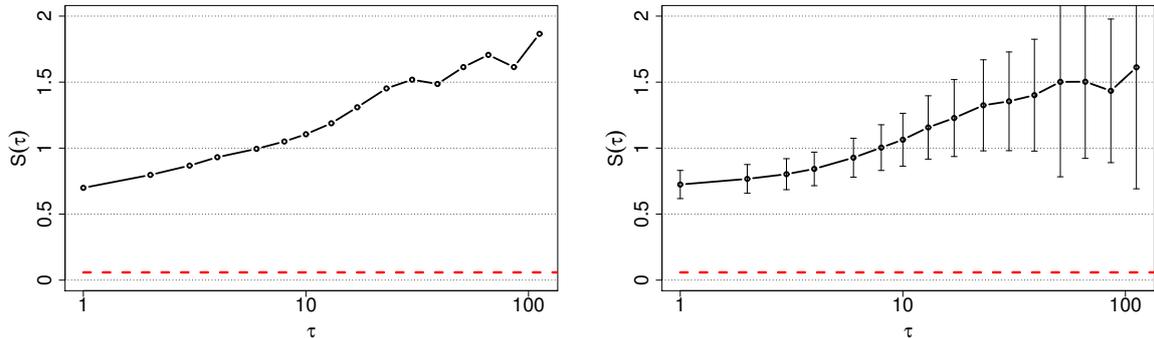


Figure 6: The splitting ratio – Splitting becomes steadily more dominant over herding as time increases. *Left panel:* The splitting ratio  $S(\tau) = C_{split}(\tau)/C(\tau)$  for the first half of 2009 for AZN. *Right panel:* The splitting ratio averaged over all 102 samples. The bars represent standard deviations. The red dashed line corresponds to the behavior of the herding null model given in Eq. (7) with parameters matched to the data.

the predicted value, which is roughly 0.06 independent of  $\tau$ . Thus the order splitting component is a factor of 10 – 30 higher than predicted.

## 2. Can the splitting null model explain the data?

So far we have compared the data to the herding null models and show they cannot explain the data. But what about the splitting null model? As already discussed in the introduction, the order splitting model of Lillo, Mike, and Farmer (2005) is capable of matching the empirical autocorrelation  $C(\tau)$  quite well. The splitting ratio of this model is  $S(\tau) = 1$ , independent of  $\tau$  and independent of the brokerage map. This ratio is somewhat too high for  $\tau \leq 5$ , where  $S \approx 0.7$ . This could be due to one of two reasons, that we cannot distinguish: (1) Some short term herding behavior or (2) partially dynamic brokerage choice<sup>24</sup>. For  $\tau \geq 10$  the splitting ratio  $S(\tau) = 1$  predicted under the order splitting scenario is actually too low, due to the negative contribution of the herding component. Thus, the order splitting model explains the basic fact of the dominance of order splitting, but to get a better match to the data one would need to include other effects, such as a small component of herding, allow of the possibility of negative contributions to herding, and possible include partially dynamic brokerage (i.e.

<sup>24</sup>By partially dynamic brokerage choice we mean that some investors (but not others) might randomly vary their brokers, or investors might have partial allegiance to a small set of brokers, randomly choosing between them but avoiding the others. As the magnitude of the random choice increases so does the herding component, and as we know from the previous scenario, unless a single brokerage dominates the trading (which is not the case for the real data), this strongly distorts the decomposition in favor of herding.

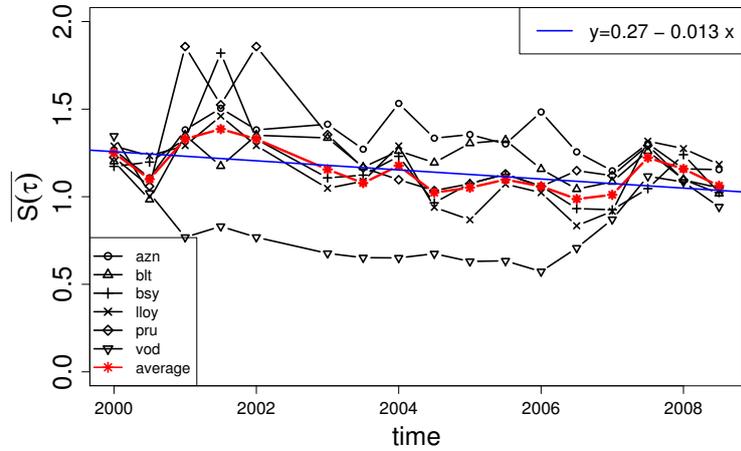


Figure 7: The evolution of order splitting versus herding throughout the decade. For each six month period we compute the splitting ratio  $S(\tau) = C_{split}(\tau)/C(\tau)$  averaged over  $1 \leq \tau \leq 50$  for each of the stocks in our data set. The red line with stars shows the average of the splitting ratios in each period, and the blue solid line is a linear regression. The regression is negative, suggesting the dominance of splitting over herding is actually decreasing slightly through time (and in any case is clearly not increasing).

to allow investors to split their trading across a few brokers, which gives the appearance of herding).

### 3. The evolution of splitting versus herding through the decade

During the last decade there has been an enormous rise in the use of algorithmic brokerages, which take large institutional orders, split them into small pieces and execute them incrementally. One might expect this activity to be reflected in an increase in order splitting over the course of our ten year data set, which covers 2000 - 2009.

To investigate this we compute the splitting ratio  $S(\tau) = C_{split}(\tau)/C(\tau)$  for each of the stocks in our data set in each half-year. The result is shown in Figure 7. The surprising result is that we observe no increase in the average splitting ratio with time – in fact, a linear regression indicates a slight decrease. The only possible exception is Vodafone, which shows an increase from 2006-2009 (but experienced a substantial decrease during the first two years). These results indicate that, at least in comparison to herding, order splitting was a common activity even before the widespread use of algorithmic trading, and that the rise of algorithmic trading has been a relatively small effect.

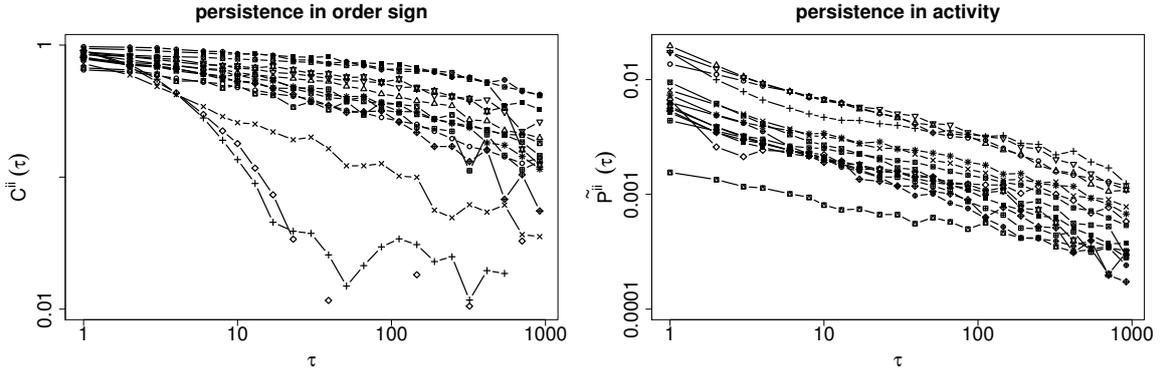


Figure 8: Heterogeneity of the contribution of individual exchange members (labeled by  $i$ ) in the diagonal component of the autocorrelation of order flow. Following the decomposition of Eq. (2), on the left we plot  $C^{ii}(\tau)$ , which measures persistence in trading direction and on the right we plot  $\tilde{P}^{ii}(\tau)$ , which measures persistence in activity. We show data for the 15 most active members for AZN for the first half of 2009, using distinctive symbols to indicate each member (consistently in both plots). For one of the members (labeled by diamonds)  $C^{ii}(\tau)$  decays more quickly than the others and for some values of  $\tau > 10$  is negative, hence the diamonds cannot be plotted on logarithmic scale and cannot be connected. This member is the fifth most active.

#### 4. Heterogeneity of individual members

In this section we study the behavior of individual exchange members and show that their behavior is remarkably consistent, with a few exceptions. The decomposition of Eq. (2) makes it clear that persistence in order flow is driven by two factors: persistence in order sign, measured by  $C^{ij}(\tau)$ , and persistence in activity, measured by  $P^{ij}(\tau)$ . Because they appear multiplicatively, both factors are needed to get a large autocorrelation. Insofar as splitting is dominant over herding the diagonal terms  $C^{ii}(\tau)$  and  $\tilde{P}^{ii}(\tau)$  will dominate the contribution due to the off-diagonal terms.

To understand how these vary across the members of the exchange, in Figure 8 we show  $C^{ii}(\tau)$  and  $\tilde{P}^{ii}(\tau)$  for the 15 most active members.

For  $\tilde{P}^{ii}(\tau)$  all of the members show strikingly similar behavior; the individual curves are fairly straight and parallel to each other. Since we are plotting the data on double logarithmic scale, the straightness of the lines indicates that the individual exchange members are all reasonably well approximated as a power law, and the fact that the lines are parallel indicates an exponent that is independent of the exchange member. The offsets indicate differences in scale. Thus from the point of view of patterns of trading activity, all members behave in more or less the same manner.

The patterns for trading sign are also consistent in most cases, but nearly so much so as activity.  $C^{ii}(\tau)$  decays very slowly, but the curves tend to steepen slightly with increasing  $\tau$ . There are three members for which  $C^{ii}(\tau)$  decays faster than the others, and

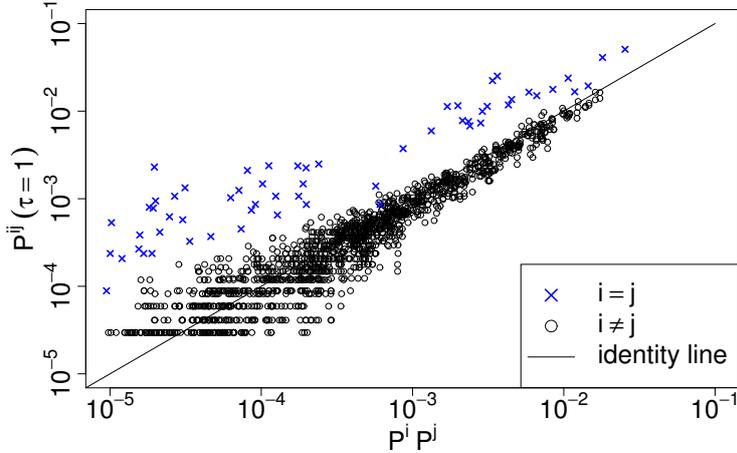


Figure 9: Persistence in activity. The joint probability of activity  $P^{ij}(\tau = 1)$  as a function of  $P^i P^j$ , the hypothetical joint probability under the assumption of independence. Each symbol indicates a given pair of members  $i$  and  $j$  for AZN during the first half of 2009. Diagonal elements  $i = j$  are indicated by blue crosses and off-diagonal elements  $i \neq j$  by black circles. The plot is on double logarithmic scale. For comparison the identity line  $P^{ij}(\tau) = P^i P^j$  is shown as a solid black line. The fact that the diagonal elements  $P^{ii}(\tau)$  consistently lie well above the identity line is consistent with the large contribution of splitting. In contrast the off-diagonal elements  $P^{ij}$  (with  $i \neq j$ ) tend to cluster along or even slightly below the identity line.

two members in particular for which the behavior is dramatically different, suggesting that these members follow a different business model.

The results in the right panel of Figure 8 indicate that the activity clustering of all members behaves similarly except for scale. The scale varies by nearly an order of magnitude, and is strongly correlated with the trading volume of the member. The Spearman rho of  $P^i$ , which measures the trading volume, and  $\tilde{P}^{ii}(1)$ , which measures the persistence, is typically very high, for example 0.92 for AZN in the first half of 2009. Because  $(P^i)^2$  is subtracted in computing  $\tilde{P}^{ii}$ , such a large correlation is not automatic. This indicates that the most active members also tend to trade in a more clustered manner than the less active firms, i.e. if they become active, they tend to remain active, and vice versa. In contrast  $C^{ii}$  and  $P^i$  are essentially uncorrelated, as are  $C^{ii}(1)$  and  $\tilde{P}^{ii}(1)$ , indicating that trading direction and trading activity are uncorrelated.

A different method of visualizing the heterogeneity of trading activity is presented in Figure 9. We plot  $P^{ij}(\tau = 1)$  against  $P^i P^j$  for all pairs of members  $i$  and  $j$ . If there is no coordination between the activity of different brokers, we expect the off-diagonal elements  $i \neq j$  (shown as black circles) to be close to the identity line. This is indeed precisely what is observed; indeed the off-diagonal elements are somewhat below

the identity line, corresponding to the negative contribution of herding. The diagonal elements  $i = j$ , in contrast, are well above the identity (shown as blue x's). This shows that there is strong persistence in the activity of brokers, but little coordination between the activities of different brokers. Larger values of  $\tau$  show similar behavior though with somewhat smaller amplitude.

These results indicate that the persistence of order flow stems from the persistence in both trading direction and activity of individual members of the exchange. They also show that the dominance of splitting over herding is also apparent in the activity level  $P^{ij}$ , where we see that the diagonal elements  $P^{ii}$  dominate over the off-diagonal elements  $P^{ij}$  with  $i \neq j$ .

## VI. Clues about the cause of the negative contribution of herding

As we have shown in Section 1, the herding component of the order flow autocorrelation is often negative for  $\tau > 10$ . This implies that buying by one investor tends to invoke subsequent selling by other investors. In this section we first test the statistical significance of this phenomenon and then perform some empirical investigations that give a clue as to its origin.

### 1. *The negative contribution of herding is statistically significant*

To assess whether the negative contribution of herding is statistically significant we perform a one-sided significance test by comparing the real data to the null hypothesis that both the order signs and brokerage codes are assigned randomly. Realizations of this null hypothesis are obtained by randomly shuffling both the signs and brokerage codes. For each of the 102 datasets we produced  $10^6$  realizations of the null hypothesis, and for each realization we computed the splitting and herding components of the autocorrelation. Then for each lag  $\tau$  we estimated the fraction of random realizations having a herding component smaller than the value observed in the corresponding real sample. If this fraction is smaller than 5%, we reject the one-sided null hypothesis. Figure 10 shows the fraction of sets for which we reject the null hypothesis. For small values of  $\tau$  we never reject the hypothesis because the real herding component of the autocorrelation function is significantly positive – this is expected since we do not observe negative contributions in this region. For values of  $\tau$  between roughly 15 and 80 we reject the null hypothesis in approximately 80% of the sets. This means that for these lag values the negativity of the herding component of the autocorrelation is statistically significant. We now explore the possible origin of this phenomenon.

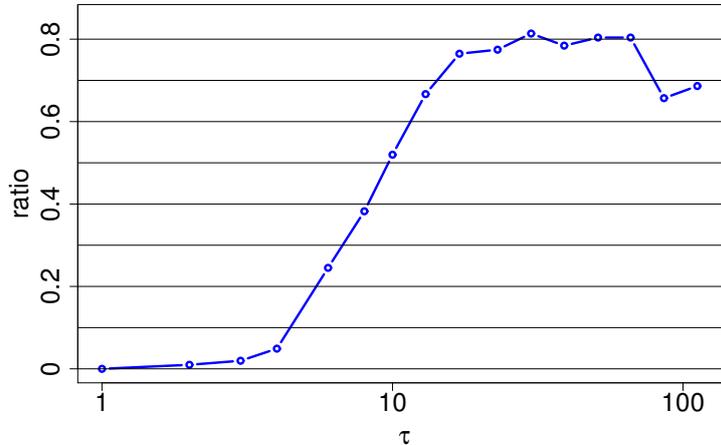


Figure 10: Statistical significance of the negative contribution of herding. The vertical axis plots the fraction of the 102 data sets for which the herding component of the autocorrelation  $C_{herd}$  has a  $p$  value less than 5% under the IID null hypothesis as a function of the lag  $\tau$ .

## 2. What underlies the negative contribution of herding?

We now show that the negative contribution of herding is associated with a difference in the response of brokers to market orders, depending on whether or not the order changes the price and whether or not it is from the same or a different broker. If a market order placed by broker  $i$  changes the price, broker  $j$  is less likely to place market orders in the same direction, while the behavior of broker  $i$  is unchanged<sup>25</sup>.

We use the notation  $MO_t^0$  for a market order at time  $t$  that does not change the price and  $MO_t'$  for a market order that changes the price. Conditioned on either of these events, the probabilities for subsequent market orders to have the same sign are

$$\begin{aligned} P(\epsilon_t = \epsilon_{t+\tau} \mid MO_t^0) \\ P(\epsilon_t = \epsilon_{t+\tau} \mid MO_t') \end{aligned} \tag{10}$$

Assuming that on average buy and sell trades have the same size and that brokers' inventories are bounded, these probabilities for large  $\tau$  should converge to the unconditional probability  $P(\epsilon_t) = 1/2$ . In Figure 11 these are shown for AZN, plotted as a function

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<sup>25</sup>The notion that whether or not a market order changes the price is an important factor was inspired in part by a previous study by Eisler, Bouchaud, and Kockelkoren (2011b); see also Toth et al. (2012).

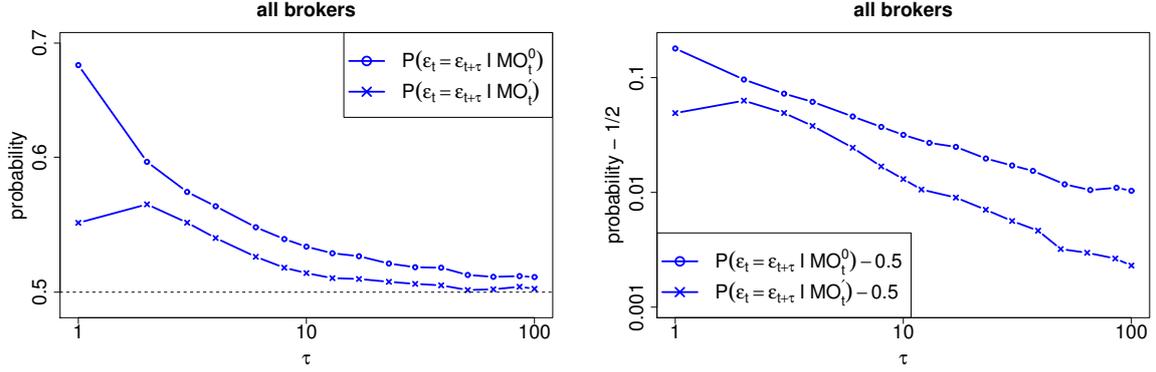


Figure 11: Probability that market orders at times  $t$  and  $t + \tau$  have the same sign, conditioned on whether or not the first market order changed the price. The case of no price change is given by  $P(\epsilon_t = \epsilon_{t+\tau} | MO_t^0)$  (represented by circles) and the case of a price change by  $P(\epsilon_t = \epsilon_{t+\tau} | MO_t^1)$  (represented by "x"). The plots on the left and right are the same, except that on the right we subtract the unconditional probability  $P(\epsilon_t) = 1/2$  to make the functional dependence on time and the difference in the convergence rates clearer. When the original market order changes the price, the decay is much faster. Data is for AZN for 2000-2009.

of  $\tau$  on a double logarithmic scale. To make the  $\tau$  dependence clearer, we also plot the excess probabilities

$$\begin{aligned}\tilde{P}^0(\tau) &\equiv P(\epsilon_t = \epsilon_{t+\tau} | MO_t^0) - 1/2 \\ \tilde{P}^1(\tau) &\equiv P(\epsilon_t = \epsilon_{t+\tau} | MO_t^1) - 1/2.\end{aligned}$$

The decay of both  $\tilde{P}^0$  and  $\tilde{P}^1$  is approximately a power law. However, when the original market order changes the price, the decay is faster. This means that, all else being equal, when a market order does not change the price the persistence of the sign of subsequent orders is much stronger than when it does change the price.

We now study how the identity of the broker affects this behavior. We look at the probability that the signs of the orders at  $t$  and  $t + \tau$  are the same, conditioned on whether they were placed by the same broker and also on whether the event at  $t$  changed the price or not. We study the following probabilities:

$$\begin{aligned}P(\epsilon_t^i = \epsilon_{t+\tau}^j | i = j ; MO_t^0) & \quad P(\epsilon_t^i = \epsilon_{t+\tau}^j | i \neq j ; MO_t^0) \\ P(\epsilon_t^i = \epsilon_{t+\tau}^j | i = j ; MO_t^1) & \quad P(\epsilon_t^i = \epsilon_{t+\tau}^j | i \neq j ; MO_t^1)\end{aligned}\tag{11}$$

In Figure 12 we plot the probabilities above on a double logarithmic scale. When the broker at time  $t$  and  $t + \tau$  is the same there is no qualitative difference, regardless of whether the order at time  $t$  changed the price, other than a small shift in scale.

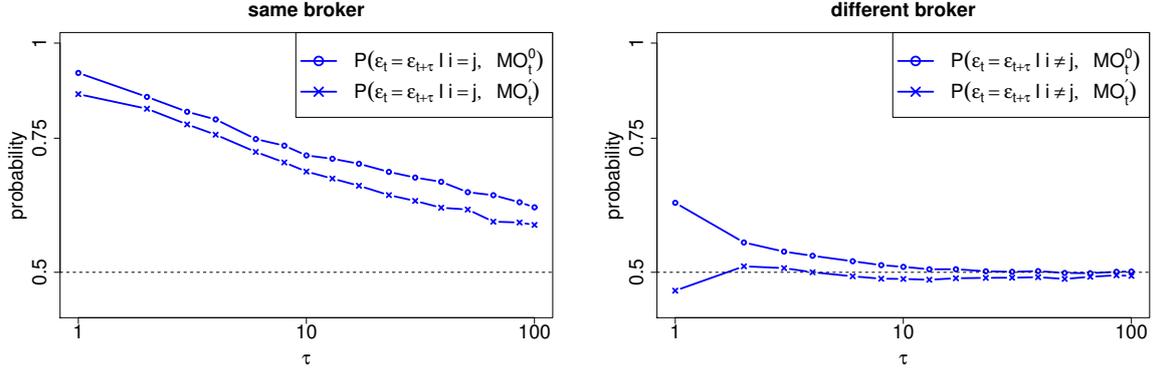


Figure 12: The probability that market order signs separated by lag  $\tau$  are the same, conditioned on whether the members placing the orders are the same and whether or not the first market order changes the price, as described in Eq. (11). *Left panel:  $i = j$ : The same member places the order at  $t$  and  $t + \tau$ .* If the market order at time  $t$  changes the price ( $MO_t^1$ , represented by "x") the behavior is nearly the same as if it does not change the price ( $MO_t^0$ , represented by "o"), other than a slight shift in scale. *Right panel:  $i \neq j$ : Different members place the order at  $t$  and  $t + \tau$ .* Whether or not the market order at time  $t$  changes the price now makes a big difference in the behavior. After an event that does not change the price, other brokers tend to put their order in the same direction with a probability higher than 0.5. After an event that changes the price, in contrast, at most lags it is slightly more probable that other brokers put their orders in the opposite direction. Data is for AZN for 2000-2009.

In contrast, when different brokers place the orders at time  $t$  and  $t + \tau$ , the behavior changes dramatically. If the market order at time  $t$  does not change the price, the probability that the signs of the two orders are the same is lower than before, but still higher than 1/2. In contrast, if the market order at time  $t$  changes the price, for most lags  $\tau$  the subsequent order is more likely to have the opposite sign. Thus, after a market order that does not change the price other brokers tend to place their orders in the same direction, indicating a slight herding on short time scales, but after a market order that changes the price, the opposite happens: they are more likely to put their orders in the opposite direction, causing negative autocorrelations.

This is confirmed by decomposing the autocorrelation conditional on whether or not the market order at time  $t$  changed the price, shown in Figure 13. More precisely, we apply the herding/splitting decomposition of Eq. (4) to the conditional autocorrelations  $E[(\epsilon_t - \mu)(\epsilon_{t+\tau} - \mu)|MO_t^0]$  and  $E[(\epsilon_t - \mu)(\epsilon_{t+\tau} - \mu)|MO_t^1]$ , where  $\mu$  is the mean market order sign. When the market order at  $t$  does not change the sign, the herding component decays to zero but never becomes very negative; in contrast, when the market order at  $t$  changes sign, it is negative for roughly  $\tau > 5$ . The shape of the herding term is qualitatively similar to the probabilities shown in Figure 12.

The conclusion is that the observed negative contribution of herding is related to the difference in the response of brokers to market orders placed by *others*, depending on

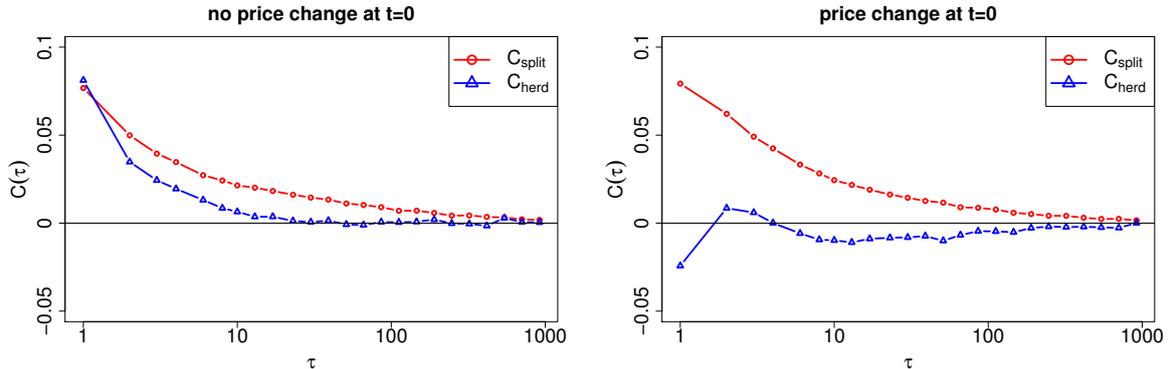


Figure 13: Splitting and herding component of the autocorrelation function of market order signs conditional on the event at time  $t$ . The left panel is conditioned on the case where the original market order does not change the price, and the right panel is the case where it does. The negative contribution to the herding term is entirely due to the latter case. Data is for AZN for 2000-2009.

whether or not they changed the price. When a market order does not cause a price change brokers continue being more likely to place orders of the same sign, regardless of who placed the original market order. In contrast, if a market order triggers a price change, other brokers place fewer market orders in the same direction than in the opposite direction. The interesting point is that splitting traders seem to act like “noise traders”, i.e. they do not adapt their behavior to their own impact. The reason for this could be that they have already calculated their impact in their estimates of the trading cost, so it is expected. The same effect is also mentioned by Toth et al. (2012).

## VII. Conclusions

We have shown that in the LSE the cause of the extreme persistence of order flow on short timescales (corresponding to 500 transactions or less, typically about an hour) is overwhelmingly due to autocorrelated trading by single investors (which we call splitting), rather than correlated trading by different investors (which we call herding). We observe only weak herding at short time scales, and for longer times we often observe that the order flow of a given exchange member is actually anti-correlated with that of others. We should make the caveat that we are only able to obtain statistically significant results for fairly short time scales of less than a few hours. It is quite possible that herding is a stronger factor on longer timescales. Nonetheless, we expected to find considerably more herding behavior than we find here.

Our analysis is based on a decomposition of the autocorrelation function into a component due to cross-correlation of different investors and a component due to autocorrelation of single investors. We apply this to data identifying the members of the

exchange, who often act as brokers for other investors. In order to understand whether these results apply to single investors we developed a set of models for investor behavior and choice of broker. We combine these into scenarios with different combinations of investor behavior and brokerage, and use these as null hypotheses. The decomposition of order flow under these null hypotheses is then compared to the data. Even under fairly extreme assumptions it is impossible to match the data with any of the null models based on herding. Our main conclusion is that, even though our analysis was done at the level of brokers, the dominance of splitting over herding applies at the level of single investors as well.

The methods that we have developed here have implications beyond this study. Data with brokerage information is more widespread than data about single investors. The methods that we have developed here provide a proof of principle for how brokerage data can be used to infer information about single investors. For example, we show that under extreme assumptions about brokerage, such as dynamically random choice of brokers, the decomposition of the autocorrelation function of order flow strongly favors herding, irrespective of investor behavior. The fact that the brokerage data strongly favors order splitting over herding shows that real brokerage choices must be very different than the dynamically random model. Instead the data are consistent with the idea that investors use only a small number of brokers and are fairly consistent in their choice of brokers through time.

Our results here are consistent with the hypothesis that the origin of long memory in order flow is mainly due to single investors who consistently execute their large orders through at most a few brokers, splitting them into small pieces. This is exactly what one expects from algorithmic execution engines, who take large orders, split them into pieces and execute them throughout the day through their own brokers. Interestingly, however, despite the increase in usage of algorithmic brokers, we do not observe any increase in order splitting as opposed to herding during the ten year period of our study. The behavior is extremely consistent across different stocks and time periods. It is also fairly consistent across member firms. While a few member firms have less directional persistence than others, the vast majority are quite persistent, and in an almost identical way. Furthermore the persistence exhibits itself similarly in both trading direction and trading activity.

As discussed in the introduction, an important consequence of autocorrelated order flow is its effect on market impact. As shown by Farmer et al. (2011) this can result in rather precise (and hence sharply testable) predictions for the functional form and dynamic behavior of market impact. For this the origin of the persistence of order flow matters: If it were due to herding the implications would be substantially different.

Under the interpretation that the strong positive autocorrelation of investors is due to order splitting, the fact that investors split their orders so strongly implies that typically the market is in a certain sense out of equilibrium. That is, if investors revealed

their intentions when they made decisions, rather than concealing their intentions and revealing them only gradually, prices at any given moment might be substantially different than observed prices. Understanding the implications of this remarkably robust and widespread phenomenon deserves closer attention.

## VIII. Acknowledgments

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## Appendix

### *Derivation of autocorrelation decomposition*

The derivation is a trivial decomposition of the autocorrelation function into its pieces when the order flow can be labeled by the identity of the agent  $i$  placing the order.

$$\begin{aligned} C(\tau) &= \frac{1}{N} \sum_t \epsilon_t \epsilon_{t+\tau} - \left( \frac{1}{N} \sum_t \epsilon_t \right)^2 \\ &= \frac{1}{N} \sum_t \sum_{i,j} \epsilon_t^i \epsilon_{t+\tau}^j - \left( \frac{1}{N} \sum_t \sum_i \epsilon_t^i \right)^2. \end{aligned}$$

Since by definition  $P^i = N^i/N$  and  $P^{ij} = N^{ij}/N$ , by interchanging the order of sums this can be rewritten as

$$\begin{aligned} C(\tau) &= \sum_{i,j} P^{ij}(\tau) \left[ \frac{1}{N^{ij}(\tau)} \sum_t \epsilon_t^i \epsilon_{t+\tau}^j \right] - \left( \sum_i P^i \frac{1}{N^i} \sum_t \epsilon_t^i \right)^2 \\ &= \sum_{i,j} \left( P^{ij}(\tau) \left[ \frac{1}{N^{ij}(\tau)} \sum_t \epsilon_t^i \epsilon_{t+\tau}^j \right] - P^i P^j \left[ \frac{1}{N^i} \sum_t \epsilon_t^i \right] \left[ \frac{1}{N^j} \sum_t \epsilon_t^j \right] \right). \end{aligned}$$

The autocorrelation function can then be written

$$C(\tau) = \sum_{i,j} P^{ij}(\tau) C^{ij}(\tau) + \sum_{i,j} \tilde{P}^{ij}(\tau) \mu^i \mu^j.$$

### *Decomposition of herding model I: Public information*

Since the investors are chosen randomly the timing and sign of the trade are independent of the investor, which implies

$$\begin{aligned} C^{ij}(\tau) &= C(\tau), \\ P^{ij}(\tau) &= P^i P^j. \end{aligned}$$

The second of these relations implies that  $\tilde{P}^{ij}(\tau) = 0, \forall i, j$ . Eq. (4) then implies that

$$\begin{aligned} C_{split}(\tau) &= \sum_i P^{ii}(\tau) C^{ii}(\tau) = C(\tau) \sum_i (P^i)^2 \\ C_{herd}(\tau) &= \sum_{i \neq j} P^{ij}(\tau) C^{ij}(\tau) = C(\tau) \sum_{i \neq j} P^i P^j. \end{aligned}$$

Substituting using the definition of the cross-sectional variance in Eq. (8) gives Eq. (7).